The Axiom of Dimensionality

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Abstract

The dimension of a space reflects the diversity of its inhabitants, the degrees by which they may move, expand, contract, grow or vibrate, but does each degree of motion have to be confined to variation along a one-dimensional parameter. Is there an implicit axiom hidden with the practice of Euclidean geometry, an axiom which, if changed and it's consequences explored, could give rise to a new creative vision of geometry.

1 Introduction

Recently I read a fascinating description of the most recent developments in String Theory, the *Theory of Everything*, the holy grail of an intellectual and logical description of the nature and evolution of our, and possibly, other universes. From the outset, science has concerned itself with developing a systematic language for elucidating the dynamic interactions which govern our view of the Cosmos. Since the time of the ancient Greeks, and perhaps even further back, this language has changed and evolved to suit experimental, theoretical and philosophical norms, and this continuous adaptation has underpinned some of the startling successes which have given rise to our current comprehension of the nature of the Universe within which we see the Laws of Nature performing their daily dance.

The elegant description of this dance published in Green's [3] book, illustrates clearly the prominent role of geometry in current developments in theoretical physics. According to the most recent research we live in a universe, which may most easily be described as a 10 or possibly 11-dimensional mathematical space-time continuum, in which all but three spatial and one temporal dimensions are confined to a (currently) non-observable region. Since there is currently no experimental means of determining the precise nature of the multidimensional mathematical space-time continuum within which our Cosmos is presumed to exist, researchers tend to rely on principles of symmetry and logic to guide their developments.

This mathematical universe is inhabited by many wonderful and exotic species, such as one-dimensional strings vibrating in six-dimensional curled Calabi-Yau spaces. More recent developments prompted by the so-called second super-string revolution has given rise to the even more exotic *p-branes*, where the one-dimensional strings have been subsumed into a more general conception of vibrations in many dimensions. It was while reading about *zero-branes*, objects which at large distances behave like point particles, but apparently have a very definite structure at the smaller distance scales within the realm of string-theory, that I was struck by an intriguing correlation. There was another version of geometry that I had explored, where points have a certain structure, and this is what leads me to discuss a fundamental aspect of geometry, which to my knowledge has never received or perhaps warranted serious scrutiny because it is a hidden axiom, I call it, *"the axiom of dimensionality"*.

2 Euclidean Dimension

When we wish to describe mathematical interactions a fundamental step in this process is to prescribe in exact a form as possible the arena within which these interactions take place. Therefore the first line in any mathematical treatise often follows the convention, "consider the set of ...", or "consider a ..." where the ellipses ... usually corresponds to a well defined, or previously prescribed group of mathematical objects which may be operated on to produce a new set of objects, and thereby develop a particular theorem or thesis.

In this instance we shall not be so precise in our prescription, except to state that geometry has as it's arena a multidimensional continuum, which may be categorised in a number of different ways. One of the most important categorisations describes space as an n-dimensional continuum with each element of the space requiring an n-tuple of independent real numbers for it's complete prescription. In addition some form of global reference point at which each of the n-tuple values is 0, is also prescribed as the origin for that space. In this way the dimensionality of our space is defined as being composed as n copies of a 1-dimensional or linear space. This categorisation of space has given rise to very powerful fields of mathematical computation, but the full effects of this definition have not yet been fully explored.

For instance there appears to be very little on the role of dimension in relation to a particular domain, save for the recognition of results on the cardinality of the dimensions of subspaces. The primary reason for this is that the independent linear spaces which go to make up the dimensional space have such a simple structure, being in one way or another isomorphic to the real line. Even though the bases for our multidimensional continuum may themselves be described as, for instance, functions they are still considered to be monadic, and having a single non-synthetic character. This prescription has served mathematics very well, but it is not the only way in which the dimensions of a space may be prescribed, and an alternative outlined below may lead to a very rich source of new computational methods and techniques in mathematics, and in geometry in particular.

3 Vedic Dimension

The previous prescription of the concept of dimension ensures that the primary distinctive feature of spaces with a similar structure relates to the cardinality of the dimensional spaces.

Each dimensional space is a linear extension of a monad, relying on transformations of linear Boolean logic. A simple change in the conception of dimension leads to a rich flowering of potential mathematical structures. This change has been developed by Kapoor [4], where he changes the hidden *axiom of dimensionality,* that "the dimensionality of each dimensional space is a linear one-dimensional monadic space", with the simple vedic concept for dimensionality that, "the order of the dimensional space is two less than the order of the domain space." Thus if we are considering a domain V_n, i.e. a multidimensional mathematical continuum with vedic dimension n, then it's dimensional spaces are characterised as $n \times V_{n-2}$, i.e. n autonomous spaces each of dimensionality $n-2$.

The rationale for making such a simple change in the conceptual prescription of dimension is discussed in detail by Kapoor [5], which expands on the detailed correlation between the various structures of multidimensional real spaces and their depiction in various aspects of the Vedic literature. In summary from his analysis of the Vedic literature Kapoor has deduced a mani-fold structure for each domain space, corresponding to a dimension-fold, boundary-fold, domain-fold and origin-fold. This is most easily observed and developed if we consider the representative regular bodies for each space, and their inter-relationships.

3.1 Representative Regular Body

When we consider a one-dimensional space, the most natural representation for this space in in terms of a unit interval, similarly for a two-dimensional space the most natural representation is a unit square, and for three-dimensions the representation is a unit cube. Each of these representative regular bodies can be derived in a sequential fashion as the track of a lower dimensional body moving in a higher dimensional space..

Thus starting with a zero-dimensional point, its track corresponds to a one-dimensional interval, moving which produces a two-dimensional square, whose movement produces a three-dimensional cube. In each case the movement is along a mathematical dimension which is not currently covered by the extension of the regular body. This successive generation of one-regular body from another at lower dimension is depicted in Table 1, where it is assumed that a is the unit of length, through which the representative regular body is moved in order to generate it's successor.

We also note that the representative regular bodies can be represented successively by the algebraic forms,

$$\begin{aligned} r_0 &= a^0 && 0 - \text{dimensional interval/point} \\ r_1 &= a^1 + 2a^0 && 1 - \text{dimensional interval/line} \\ r_2 &= a^2 + 4a^1 + 4a^0 && 2 - \text{dimensional interval/square} \\ r_3 &= a^3 + 6a^2 + 12a^1 + 8a^0 && 3 - \text{dimensional interval/cube} \end{aligned}$$

The question now arises as to what is the geometrical structure of a representative regular body of four-space, and how may it be generated.

Our first clue here is that the representative regular body of 4-space is a hyper cube generated by moving our 3-cube, or the geometric body corresponding to the algebraic expression

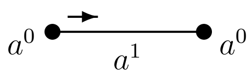
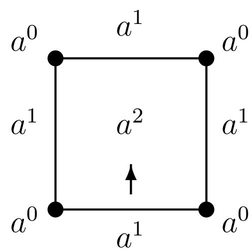
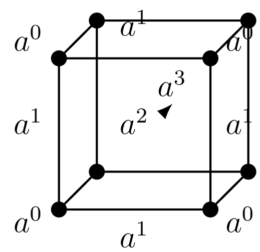
1-space body	2-space body	3-space body
		

Table 1: Representative Regular Bodies

$r_3 = a^3 + 6a^2 + 12a^1 + 8a^0$, along a mathematical hyperspace dimension which is distinct from each of the currently visible spatial dimensions. As with each of the previous generation steps, we can identify the content contributed to the hyper-space cube by each of the constituents of the 3-cube. Thus each zero dimensional point, manifests a 1-dimensional interval in addition to two boundary points, each 1-dimensional interval manifests a 2-dimensional square, in addition to two boundary intervals, and each square manifests a 3-dimensional interval in addition to two boundary squares. This manifestation process may be summarised as,

$$\begin{aligned} a^0 &\xrightarrow{M} a^1 + 2a^0 \\ a^1 &\xrightarrow{M} a^2 + 2a^1 \\ a^2 &\xrightarrow{M} a^3 + 2a^2 \end{aligned}$$

based on which we can stipulate that the manifestation product of a 3-dimensional cube corresponds to a 4-dimensional interval, in addition to two boundary cubes. Thus the manifestation of the 3-cube content may be represented algebraically as

$$a^3 \xrightarrow{M} a^4 + 2a^3$$

In order to determine the complete form for the representative regular body corresponding to a 4-dimensional hyper-space it is necessary to account for the contribution of each constituent of the 3-dimensional cube. This may most easily be accomplished by considering the algebraic representation r_3, and noting that the manifestation of the constituents along the hyperspace is equivalent to multiplication of each constituent by $a + 2$, or

$$r_3 \xrightarrow{M} r_3(a+2) \quad \Rightarrow \quad r_4 = a^4 + 8a^3 + 24a^2 + 32a^1 + 16a^0$$

which gives an algebraic form for the hyper-space cube generated by moving a 3-cube in a mathematical hyper-space dimension independent from the existing visual dimensions.

A comprehensive analysis of higher-dimensional spaces as represented in the Vedic literature has allowed Kapoor [4], to infer that this particular generational form is also consistent for the generation of higher-dimensional regular representative bodies, from which we can infer the following relationship,

$$r_{n-1} \xrightarrow{M} r_{n-1}(a+2) \Rightarrow r_n = (a+2)^n = \sum_{j=k}^{n} \binom{n}{k} a^{n-k} 2^k$$

which allows us to stipulate the exact constituents which compose the representative regular body corresponding to each space.

The Vedic interpretation of this expression gives rise to a continuum of manifold spaces wherein transition from one space to the next is achieved through transcendence at the origin of the domain, which is proposed to be a specialised space content with a direct link to the higher dimensional space. Thus the origin of three-space is conceived as an element of four-space and on approaching the origin, we assume transcendence into a four-space domain, with the transcendental result of being located on one of the boundary components of a four-dimensional hypercube. This contrasts greatly with the current prescription of dimension in the Euclidean sense, where all points in the domain are considered equal, and in a sense the only way to transcend to a higher dimensional space is to move the space as a whole.

Fold	I	II	III	IV
Type	Dimension	Boundary	Domain	Origin
Order	$n-2$	$n-1$	n	$n+1$

Table 2: Manifold Prescription for Vedic Domain of Dimension n

The inclusion of the origin of the domain as a separate and distinct element with it's own properties leads to the manifold prescription of a mathematical continuum of spaces which are required for the complete description of any multi-dimensional space. These are depicted in Table 2, where we see the essential constituents for the space as a whole.

We see that the elements of a Vedic domain are composed of n-dimensional space content, this contrasts greatly with a Euclidean domain, where each of the elements, i.e. the points in a Euclidean domain are considered as zero-dimensional space content, with only the position of the points being prescribed as an n-tuple. Thus it is not possible to explicitly distinguish, based on internal content, between points in a Euclidean domain, or points on it's boundary. Boundary points are normally prescribed using some form of constraint equations, in other words conditions which are external to the prescription of the points themselves. In the case of a Vedic domain such a distinction is readily built into the prescription of the dimension, as the order of the space content is different both in terms of the cardinality of the dimensions of the point, and also in terms of the dimensionality of the underlying dimensional spaces.

4 Constituents of Vedic Space

When we consider the prescription of dimension in an n-dimensional Euclidean space E_n, the actual n-tuple which represents the dimension is a description of the *location* in E_n of a zero-dimensional point. It relates to the *measurement* of the n-parameters required to precisely prescribe the point.

In general according to the Vedic prescription of dimension the n-tuple of parameters used to describe an element of a Vedic space V_n are qualitatively different. That is because each of the n-tuples are themselves elements of a Vedic space V_{n-2}, a space of dimensionality $n-2$. This illustrates that in fact that whilst the elements of Euclidean space are zero-dimensional points, the elements of Vedic space have a space content structure intimately linked to the underlying dimensional space. Thus it is possible to say that elements or points in a Vedic domain V_n are a structured unity composed of n unities of lower dimension.

Let us see how this difference in prescription influences our perception of Euclidean 3-space E_3, and the Vedic 3-space domain V_3. In each case the dimensional space is linear, being composed of E_1, or the real line $\mathbf{R}$ for the former case, and being composed of V_1 in the latter. Since each point in E_3 is represented as a triple (x, y, z) of real numbers, the points belonging to E_3 are composed of the Cartesian product $E_1 \times E_1 \times E_1$. In a similar manner the elements of V_3 may be prescribed as a triple (x^1, y^1, z^1), but in this case each element a^1 is not a real number but an extended space content of V_1. Thus although we use a notion similar to Cartesian product, the extended notion of the elements composing V_1 implies that the constituents of the Vedic domain $V_3 = V_1 \boxtimes V_1 \boxtimes V_1$ has an extended solid characterisation. Symbolically we distinguish between the Cartesian product $\times$ of Euclidean spaces, and the Tirthaji product $\boxtimes$ of Vedic domains, as exemplified in the arithmetic domain by the third of the *Ganita Sutras*, described by Tirthaji [9].

Note: It will be necessary to demonstrate a fundamental difference between the elements of E_1 and V_1, in that in the former individual elements are zero-dimensional points whereas in the latter individual elements are one-dimensional space content.

Moving to the hyperspace domain V_4 and it's comparison with *Euclidean 4-space*, we see that $V_4 = V_2 \boxtimes V_2 \boxtimes V_2 \boxtimes V_2$, which implies that the space content of the V_4 domain may be represented by a quadruple (w^2, x^2, y^2, z^2), each of whose constituents has a spatial extension being an element of V_2. This contrasts greatly with $E_4 = E_1 \times E_1 \times E_1 \times E_1$, where the elements of E_4 are represented by the quadruple (w, x, y, z) of real numbers.

5 Constructing the Hyper-Circle

It has taken nearly two years since writing the previous section to being able to continue.

When I began studying Vedic space, one of the aspects of it that eluded me was to develop a constructive approach for a hyper-circle, in the manner of the previous section for a hyper-cube, which is not my construction but is due to Dr. Kapoor. The aspect which particularly eluded me was the concept of the metric in V_4. In terms of Euclidean spaces, since the dimension is linear, the simplest metric is used to determine the construct of the hyper-circle, let us call it C_4^E. With the Euclidean norm providing the metric. However in order to construct C_4^V, the hyper-circle in the Vedic space, V_4, this approach does not work since the dimensions are planar V_2, and it was unclear which form of metric to use, in order to develop a system for constructing C_n^V in a consistent way. Patience is not always my virtue, but as the saying goes, 'everything comes to he who waits.' In fact I hadn't waited, I had forgotten about this paper and just opened a deleted users account, and had a poke around.

The reason I did this is that recently I have begun to resurrect research from more that 10 years ago, related to using the Irish language as a language of science and technology. This relates to the ability to represent the emergence of the quark from the unifield in terms of the phonology of certain sounds in Irish. The most significant of these is *cor*, and in my previous presentations on what I now call *Cor Ealadha*, I was able to illustrate concepts such as symmetry breaking, closed strings, quantum foam, in a phonetic rather than as a purely mathematical abstraction. This research was a wonderful journey deep into the world of the *Unified Field* guided by a wonderful book which delineates the relationship between modern science and vedic science. I was also guided by one of our own Ó Muirthile [8], which provides a rich source of authentic sounds in the Irish language, and is accompanied by two compact disks containing a recording by the author. This journey convinced me that Irish was a natural language for expressing some of the abstractions of modern physics, due to it's own flexibility, and also due to it's relationship with *Sanskrit*. I have taken up this quest again in the form of a blog which will also contain animations, to illustrate some of the concepts involved. If you have any interest in joining *Dream na nDuthoilreachta ar Seilg an gCuarc*, *The Natural Law Posse Hunting the Quark*, then you may do so at www.taristeach.blogspot.com, it promises to be a very interesting *toraiocht*.

Based on these very recent experiences I got an idea tonight for how to proceed, and develop a systematic approach for constructing hyper-circles of the form C_n^V. It involves the use of one of the *Ganita Sutras* which Dr. Kapoor has associated with reflection in a mirror. In fact I never comprehended it's potential significance until now. I'm first going to give this in word form, and then using a very small amount of Irish, introduce the potential significance to *Tensor Analysis*. It's thirty years since I did the type of *Tensor Analysis*, and I'm a little bit rusty, so please be kind in your reading, and just let it flow. I did some since as preparation for my PhD, and I know the man with that knowledge, and I hope we can resurrect it again.

When I was thinking of how to construct hyper-circles, as I mentioned previously my initial attempts were based on using an approach similar to Euclidian norm, and developing a Vedic norm. This may still be possible, but I do not think at this stage that it is the simplest approach. After much tooing and frooing. I believe that my current approach will work.

We start with a *di-monad* which consists of five distinct elements, two boundary points, two linear-space content elements, and a joint at the centre. This in fact represents C_1^V, it is a circle in a 1-dimensional space. By rotating the di-monad, through 180^o, in an axis perpendicular to the line, we obtain, C_2^V, a circle in a 2-dimensional space, consisting again of five elements. Upper and lower circum-linear boundaries, upper and lower plane-space content elements, and a rotated *di-monad*. This again shows reflection properties. Finally to obtain C_3^V, the hyper-circle in Vedic 3-space, we rotate again this time in the remaining three space dimension normal to the plane, again by 180^o, and we see the pattern repeated. There will be an upper and lower hemispherical shell, and also upper and lower three space content, and of course a twice rotated *di-monad*. That was as far as I got, until I took a little nap, and of course kept computing mentally, trying various different arrangements to transcend the apparent barrier. I then resumed the chase, and I thought of *cor*, and *cor ciall*, *cor cíl*, *circle* ... . Ah! physics and geometry meet, in phonology.

At this stage I began to wonder how to represent this algebraically and it is all in the *di-monad*. So let us say that we start with d_0^0, this is a tensor. After the first rotation the rotated *di-monad* becomes d_1^0, I cannot remember the tensor operation which is required, but I know I studied it but never knew why. After the second rotation the twice rotated *di-monad* becomes d_2^0, this produces C_3^V, a circle in a 3-dimensional space. It seems perfectly logical, in my *neuristic* way, that the way to proceed is to rotate again in V_4 to produce the thrice rotated *di-monad*, and then obtain C_4^V, the four space hyper-circle, which should again have five elements. A full description of these will require more expertise in terms of manifolds and everything else.

That's as much as I'm going to do now, as I just wanted to get this down. There are a number of important references which are available on-line. The first is www.focal.ie, a very good Translation Dictionary. The second is www.aimsigh.com, which is a door into a wonderful world of knowledge. Now, I have one reference, which seems to be quite rare, Mc Cionnath's [6] English-Irish dictionary, is a contemporary of Dineen's [1] Irish-English dictionary, and open's up a whole host of connections with our own Gaelic Literature. So you never know what we might find. Maybe even the Irish developed tensor analysis, but passed it down using the traditional teaching methods of *beal oideas*, word of mouth, teaching by mouth, phonetic precison.

5.1 Review and Revise

This gets even curiouser and curiouser.

The basic form for d_0^0 is $[a^0, a_+^1, j, a_-^1, a^0]$, where a_+^1 and a_-^1, represent directed line segments from the boundary points towards the joint j. An expression for d_1^0 is $[c_+^1, c_+^2, r[j], c_-^2, c_-^1]$, with the plus representing the upper domain, and the minus representing the lower domain. The effect of the rotation on the joint is yet to be specified. Following in this fashion an expression for d_2^0 is $[s_+^2, s_+^3, r[r[j]], s_-^3, s_-^2]$, with the plus representing the northern hemisphere and the minus representing the southern hemisphere. In terms of V_2, I use the symbol c to represent the circum-linear boundary, and the circular area, and in terms of V_3 I use the symbol s to represent the spherical-shell boundary, and the spherical volume. Now we can see the structure beginning to emerge. The numerical values indicating the dimensionality of the content and the positive and negative signs indicating the positional sign-ness. In fact from looking at the structure I would modify it to make use of covariant and contravariant tensors, and thereby represent the hyper-circle in V_4 as

$$C_4^V = [h^3, h^4, r^3[j], h_4, h_3] \tag{1}$$

with the form suggesting that the boundary consists of a three dimensional cap, enclosing a four dimensional head. This is very similar to the previous case of the development of the hyper-cube. Note also the direct relationship between cardinality of the vedic domain we are transcending and the rotation operator, in this case r^3. This illustrates one of the important keys into vedic geometry, which is the significance of cardinality.

More and more curious.

Before writing the previous paragraph I took a quick look at Feynman [2], because I knew that there was a treatment of tensors in it. Probably not as complete as the tensor course which I completed, but sufficient to get the ball rolling. Chapter 31 deals with tensors, and it is recommended to review Chapter 11, Vol. 1, *Vectors*, and Chapter 20, Vol. 2, *Rotation in Space*, a quick look at 11.3 Rotation, suggests that this is the time to introduce Williams [10], which gives an introduction to the application of vedic mathematics in astronomy, with practical applications of Pythagorean Triples. I also see that on the margin of page 20.3, beside the second last paragraph I have written *'What about real 4-space,'* so obviously I have been down this path before. Real 4-space is another name for V_4, and represents the first of the transcendental real spaces of higher dimension. Chapter 20 also happens to deal with the motion of a gyroscope, which formed a good part of my second year Mathematical Physics course. I might revisit this area again, but this time armed with Williams [11], the full blown treatment of triples. Their unique ability to unify computation of trigonometric type functions and their application in spherical geometry, could simplify an area I have been interested in since boyhood and first picked up a gyroscope.

At present I am disconnected from the net and searching for an appropriate Irish form for tensor, and the reference to hand, the Dictionary of Science, gives the word *teinseoir*, which seems like a phonetic adaptation, however in fact it may not be because I suspect that the adaptation was based on the concept of stress and tension in elasticity, and a related sound *teann* suggests narrowing. However the types of tensors that I foresee have more in common with the metric tensors used in General Relativity theory, so in time perhaps we shall discover a new Irish word for tensor.

5.2 Stitching

During my initial contacts with Dr. Kapoor, on the internet. One of the points he referred to was stitching. In the current context I believe that this is the process of knitting together the circle at the and of it's rotation. Previously I have only considered the final forms of the hyper-circles, the final shapes. I feel that there are three processes to be considered. *Tús. Deireadh. Le chéile.* Which in a different context, that of multiplying by 11 is in the form of a circle. It is necessary to carefully consider what computational steps are required at the beginning of the rotation, during the movement through a higher space, and at the end, because such transformations should be reversible. We don't want to get lost in a higher space, lost to the abstraction and unable to relate our discoveries to 3-space reality.

5.3 Adaptation

As I mentioned previously adaptation of concepts into Irish is part of the process that I am interested in. Now this is for a very specific reason. Some years ago I was working in the Squeak flavour of Smalltalk, designing a class system for numerical work. For my own reasons, I wanted to redesign integer arithmetic so that I could trap basic arithmetic operations. I created my own number class, but relatively quickly the nomenclature became very cumbersome. Then I created a class I called *uimhir*, and, to cut to the chase within three weeks I had created a tiny little *Gaeltacht*, in Squeak, where all reserved words were adapted into Irish, and I had a little interface to communicate with existing classes. The only thing I didn't manage to do was to get the *seimhiu* to work properly. I even had an Irish script. The significance of it was two-fold.

From a development perspective I would work on both systems, and if I encountered an obstacle in one, I would step out into the other natural language, and go around the obstacle. I explained this once as tunnelling, and I had encountered it when I was doing research on Special Functions, where the initial evaluation process was done on the real line. However certain functions or certain computational processes became unstable at points of discontinuity. By adapting the techniques into the complex domain, these apparent singularities could be dealt with very easily. So in this context I think of English being one axis of development, and Irish being the other. Of course the real fun happens in the plane itself.

From a production point of view developing the ability to create software in a natural language, software which can run on any machine in the world, acts as a natural copyright, which I believe would be very beneficial for the creation of a software cottage industry.

In the current context adaptation could open up a whole new source of knowledge, and I'm going to begin the process with one word which comes from Gaothán [8], and I also discovered recently on www.aimsigh.com. I had previously used this word to create an adaptation of *helicopter*, into Irish based on it's Greek roots. The word is,

> Rothlú
>
> Rothóir, Rothaireacht, Rothar

which gets us into a completely new dimension in relation to computation. I have some material on a blog which begins this process, but I don't have access to it right now.

6 Higher Space Rotations

The principle aspect of developing a rotation from a 3-space sphere into a 4-space hypersphere will be developed using advances in Tensor Calculus. Initially this will be developed for rotations from 1-space to 2-space, by using Pythagorean Triples as applied to V_2, thereby extending the work of Dr. K. Williams from spaces of Euclidean dimension, to Vedic real spaces of vedic dimension. Then we will consider rotations in 3-space, with a further extension of triples into V_3.

6.1 Rotations in Real 2-Space

The standard approach to describing rotations in a plane is via polar co-ordinates (r, θ), where r represents the distance from the origin of rotation, and θ represents the angle of rotation. A more complete approach, in my view, is to represent a rotation in terms of a triple $[x, y, r]$ with the constraints that the real numbers correspond to the sides and diagonal of a right-angled triangle, with $r^2 = x^2 + y^2$. Combined with a specialised form of triple arithmetic based on the *Ganita* sutra, **vertically and crosswise**, they may be shown to very easily compact all of the standard trigonometrical relationships normally required to describe rotations in a plane. A full and extensive treatment beginning with mental computation of triples and their arithmetic is developed by Williams [11], and is recommended for gaining a fundamental appreciation of their usage.

Triple Arithmetic

In the following I will describe triple arithmetic in algebraic terms, and see how we can extend their usage from E_2 to V_2. The first fundamental operation is of course addition. We can symbolically represent a triple as a whole as $a) = [x, y, r]$, or $a)xyr$. In order to combine two triples $a_1)x_1y_1r_1$, and $a_2)x_2y_2r_2$, in a mathematically consistent way, we define $a_1 + a_2$ as being equivalent to the triple $[x_1x_2 - y_1y_2, y_1x_2 + x_1y_2, r_1r_2]$. This may seem inordinately complex until we lay it out in a slightly different form, which exposes the **vertically and crosswise** pattern.

a_1	x_1	y_1	r_1
a_2	x_2	y_2	r_2
$a_1 + a_2$	$x_1x_2 - y_1y_2$	$y_1x_2 + x_1y_2$	r_1r_2

Table 3: Triple Addition

The first number in the triple is obtained by *vertical* addition of the x values, and *vertical* subtraction of the y values. This combination of addition and subtraction in the computational process is quiet normal and reflects the principle of combining apparent opposites to complete a process. The second number in the triple is obtained by *cross-wise* multiplication and then addition, and the final value is obtained by *vertical* multiplication. It is no harm to work through some numerical examples to fix the pattern.

Triple doubling, and triple subtraction are very easily derived from the definition of triple addition, and since we are combining right angles triangles in a particular way we are actually adding, doubling and subtracting angles, the basis for describing rotations in a plane. This becomes much clearer if we confine our triangles to the unit circle.

a	x	y	r
$2a$	$x^2 - y^2$	$2xy$	r^2

Table 4: Triple Doubling

a_1	x_1	y_1	r_1
a_2	x_2	y_2	r_2
$a_1 - a_2$	$x_1x_2 + y_1y_2$	$y_1x_2 - x_1y_2$	r_1r_2

Table 5: Triple Subtraction

Trigonometric Rules

For triples which are constrained to lie on the unit circle, we can represent them in the form,

$$\alpha) = [\cos\alpha, \sin\alpha, 1],$$

with x corresponding to the cosine of the angle, y corresponding to the sine of the angle, and the diagonal being of unit length. From this we immediately get the trigonometric formula $\cos^2\alpha + \sin^2\alpha = 1$, and can easily derive the rules for combining angles.

Addition

α	$\cos\alpha$	$\sin\alpha$	1
β	$\cos\beta$	$\sin\beta$	1
$\alpha + \beta$	$\cos\alpha\cos\beta - \sin\alpha\sin\beta$	$\sin\alpha\cos\beta + \cos\alpha\sin\beta$	1
$\alpha + \beta$	$\cos(\alpha + \beta)$	$\sin(\alpha + \beta)$	1

Doubling

α	$\cos\alpha$	$\sin\alpha$	1
2α	$\cos^2\alpha - \sin^2\alpha$	$2\sin\alpha\cos\alpha$	1
2α	$\cos(2\alpha)$	$\sin(2\alpha)$	1

Subtraction

α	$\cos\alpha$	$\sin\alpha$	1
β	$\cos\beta$	$\sin\beta$	1
$\alpha - \beta$	$\cos\alpha\cos\beta + \sin\alpha\sin\beta$	$\sin\alpha\cos\beta - \cos\alpha\sin\beta$	1
$\alpha - \beta$	$\cos(\alpha - \beta)$	$\sin(\alpha - \beta)$	1

Language Considerations

As I mentioned previously one of the aims in developing this approach to geometry is also to develop a consistent naming convention. Suggestions for some of the concepts, up till now are,

monad *aonad*

di-monad *dónad*

tri-monad *tríonad*

triple *tríocht*

plane motion *gluaiseacht dhátoise*

plane rotation *rothlú aontoise*

This list will be expanded and consolidated once more progress in developing the rotations for ascending to, and descending from, higher spaces has been achieved. It is worthwhile giving attention to language adaptation at an early stage before the mathematics obscures the simplicity. A good place to start hunting for words in Irish is *Gaoth an Fhocail* by Ó Doibhlin [7], you'd be surprised what you would find in it.

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References

[1] Dineen, P.S. *Foclóir Gaedhilge agus Béarla an Irish-English Dictionary* Dublin: Irish Texts Society, (1927) 1996.

[2] Feynman, R.P., Leighton, R.B., Sands, M. *The Feynman Lectures on Physics*, Reading, Massachussets: Addison-Wesley, 1965.

[3] Green, Brian *The Elegant Universe*, London: Vintage, 2000.

[4] Kapoor, S.K. *Vedic Geometry*, New Delhi: Arya Book Depot, (1994) 2004.

[5] Kapoor, S.K. *Fermat's Last Theorem and Higher Spaces Reality Course*, New Delhi: Arya Book Depot, 1996.

[6] Mc Cionnath, L. *Foclóir Béarla agus Gaedhilge – English-Irish Dictionary* Oifig Díolta Foillseacháin Rialtais: Baile Átha Cliath. (1935) 1948.

[7] Ó Doibhlin, B *Gaoth an Fhocail-Foclóir Analógach* Baile Átha Cliath: COISCÉIM, 1998.

[8] Ó Muirthile, L *Gaothán*, An chéad chló, Cois Life Teoranta, 2000.

[9] Tirthaji, B.K. *Vedic Mathematics*, Delhi: Motilal Banardidass, (1965) 1989.

[10] Williams, K.R.*Astronomical Applications of Vedic Mathematics*, Delhi: Motilal Banardidass, (2000) 2003.

[11] Williams, K.R.*Triples*, Delhi: Motilal Banardidass, (1984) ????.